Adaptively Exploiting d-Separators with Causal Bandits

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Regret: $$R_{\nu,\pi}(T) = T \cdot \max_{a \in \mathcal{A}} \mathbb{E}_{\nu_a}\left[Y\right] - \mathbb{E}_{\nu,\pi}\left[\sum_{t=1}^{T} Y_t\right].$$

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We introduce the *conditionally benign property*.

Definition (informal)

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UCB guarantee without any assumptions: $R_{\nu,\text{UCB}}(T) \leq \tilde{O}\left(\sqrt{|\mathcal{A}|\,T}\right)$.

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Theorem: Existing algorithms do not adapt to failure of assumptions.

For every $\mathcal{A}$ and $\mathcal{Z}$, there exists ν such that

$$\lim_{T\to\infty} \frac{R_{\nu,\text{C-UCB}}(T)}{T} \geq 1/120.$$

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Theorem: Our new algorithm HAC-UCB achieves non-trivial adaptivity.

For any $\mathcal{A}$, $\mathcal{Z}$, T, ν, and $\tilde{\nu}(Z)$,

$$R_{\nu,\text{HAC-UCB}}(T) \leq \tilde{O}(T^{3/4}).$$

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Conclusions

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- Can we exploit benign structure when it exists while still learning consistent estimates when it doesn't?
- We formalize and generalize what structure makes the task easier.
- We show existing algorithms fail strongly when assumptions fail.
- We show optimal adaptivity is impossible and formalize the Pareto trade-off for conditionally benign.
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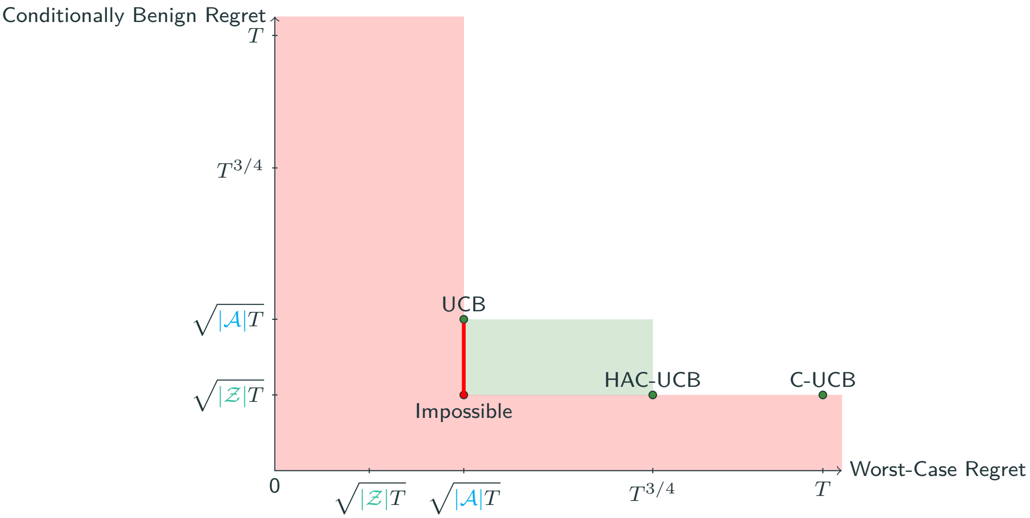

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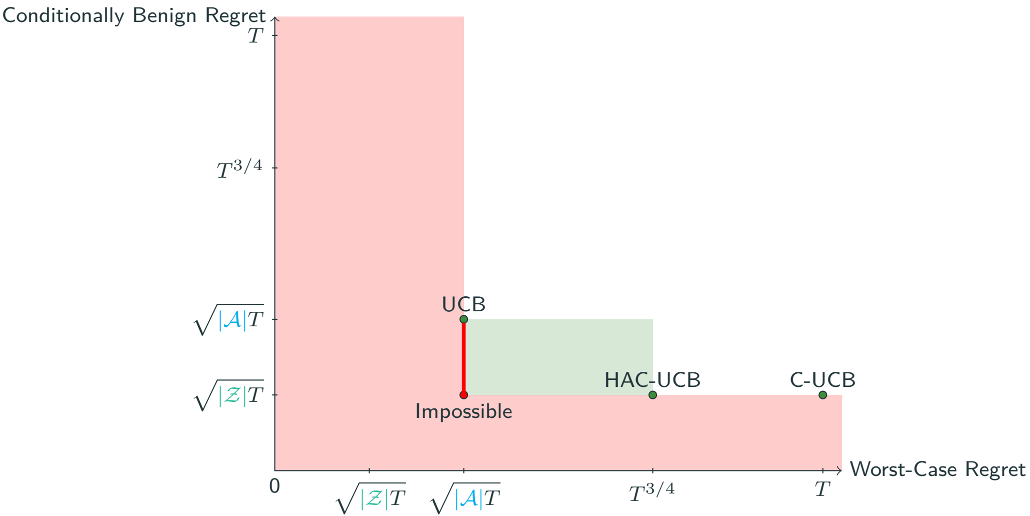

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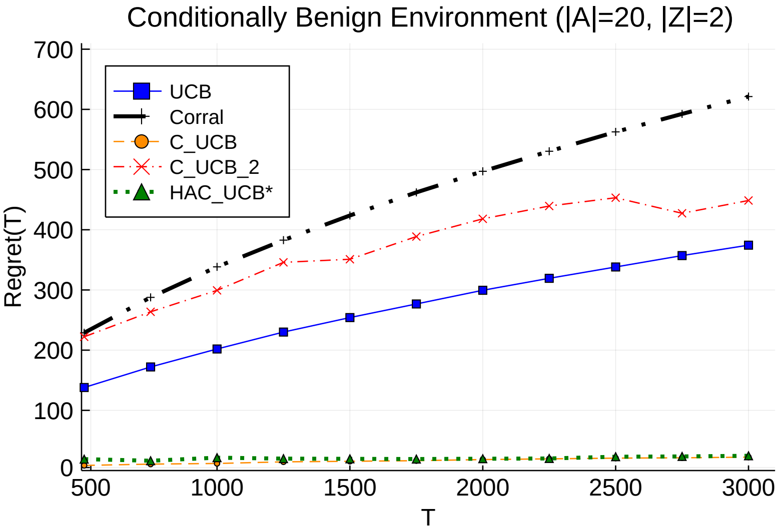

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